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Decentralized Control of Large Flexible Structures by Joint Decoupling

Tzu-Jeng Su* and Jer-Nan Juang†
NASA Langley Research Center, Hampton, Virginia 23662

This paper presents a novel method to design decentralized controllers for large complex flexible structures by using the idea of joint decoupling. Decoupling of joint degrees of freedom from the interior degrees of freedom is achieved by setting the joint actuator commands to cancel the internal forces exerting on the joint degrees of freedom. By doing so, the interactions between substructures are eliminated. The global structure control design problem is then decomposed into several substructure control design problems. Control commands for interior actuators are set to be localized state feedback using decentralized observers for state estimation. The proposed decentralized controllers can operate successfully at the individual substructure level as well as at the global structure level. Not only control design but also control implementation is decentralized. A two-component mass-spring-damper system is used as an example to demonstrate the proposed method.

Introduction

CONTROL of flexible structures has gained much research interest since the Space Shuttle transportation system became reality and construction of large structures in space is no longer a dream. Although many structural control design methods have been proposed in the past decade, the application of decentralized control to flexible structures has not been pursued extensively. A large space structure must be built incrementally with components shipped into space sequentially during several Shuttle missions. Also, for operational purposes, space structures may need to be connected and disconnected routinely in space. For these reasons, it is desirable to have controllers that can function well on each individual structure and also on the interconnected structures. Decentralized controllers designed based on structure components will meet such purpose.

Most of existing decentralized control design methods were developed for application to electrical engineering systems or economics systems, rather than to flexible structural dynamics systems (see, for example, Refs. 1–3 and Ref. 4, a special issue of *IEEE Transaction on Automatic Control* on decentralized control systems). In general, structural dynamics systems are strongly coupled (in physical coordinates), which makes it difficult to design and to implement decentralized control. There exist several decentralized control methods for flexible structures, most of which adopt or extend the concepts and methodologies developed for other type of systems. For instance, the method in Ref. 5 extends the concept of decentralized fixed modes of Ref. 1 to structural control problem. The decentralization of the control problem is based on modes instead of physical components of the structure. The controlled component synthesis (CCS) method of Ref. 6 adopts the concept of overlapping decomposition in Ref. 3 to design controllers for physical structural components. Because overlapping decomposition requires information about adjacent components, the CCS method is not a strict decentralized approach. The substructure controller synthesis (SCS) method in Ref. 7 is a decentralized control design method which uses natural decomposition of structural dynamics systems. Both CCS and SCS methods employ interface compatibility conditions to assemble the substructure controllers into a global controller for the assembled structure. However, because interface equilibrium conditions are not considered in the assembly process, the global controller does not guarantee stability of the closed-loop system, which is a major disadvantage of both CCS and SCS methods.

This paper presents a novel decentralized control design method based on the idea of joint decoupling. The method is developed specifically for decentralized control of large complex flexible structures. The structure to be controlled is decomposed into several substructures with the assumption that there is a collocated actuator/sensor pair at every interface (or joint) degree of freedom. Then, the joint actuator commands are set to cancel out all of the forces acting on the joint degrees of freedom. By doing so, the joint degrees of freedom are decoupled from the interior degrees of freedom and, therefore, there will be no interaction force transmitted through the joints from one substructure to another when the substructures are connected together. The control commands for the interior actuators (i.e., actuators that are not located at the joints) of each substructure are chosen to be localized state feedback. A similar idea is used in the derivation of a decentralized observer. The controller designed by using the idea of joint decoupling is called the *joint decoupling controller.*

The proposed joint decoupling controller has several advantages. First, the design procedure is completely decentralized in that it requires no information about the other substructures to do control design for one substructure. Second, the controllers can operate successfully at the individual substructure level before assembly and also at the global structure level after assembly. This feature makes the proposed method extremely useful for active control of space structures that need to be connected and disconnected on a routine basis. No controller redesign or controller shut-off-and-turn-on is necessary before, during, or after the connection process. Third, the proposed method divides a large-scale control problem into several small-scale subproblems, and, hence, computationally it is more efficient than a centralized control design approach. The control law can be determined by using any state feedback control design method, e.g., the linear quadratic regulator (LQR) method or pole placement method.

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*National Research Council Resident Research Associate. Member AIAA.

†Principal Scientist, Spacecraft Dynamics Branch. Fellow AIAA.

The organization of this paper is as follows. For clarity, the derivation is based on a coupled-substructure state-space model, which is obtained by enforcing interface compatibility and equilibrium conditions to substructures. The derivation of the coupled-substructure state-space model will be presented first. Then, the procedure to design decentralized control gain matrices and the procedure to design decentralized observers will be developed. A summary of design steps for the proposed joint decoupling controller is also included. Finally, a six degrees-of-freedom mass-spring-damper example is provided to demonstrate the proposed method.

Coupled-Substructure State-Space Model

This section presents the derivation of the coupled-substructure state-space model for flexible structures that are composed of many substructures. Without loss of generality, we will consider the two-substructure structure shown in Fig. 1. (The derivation for multisubstructure structure can be found in Ref. 8.) Figure 1a shows the assembled structure as an entity, which also will be referred to as the global structure. Figure 1b shows the decoupled two substructures α and β. These two substructures can be physical components that are to be connected together to form the structure, or they can be the result of applying an "imaginary cut" to the structure. Locations of actuators and sensors are denoted by u and y, which are divided into two groups. Those actuators and sensors located at the joint (or interface, where the substructures are to be connected together) are denoted with a superscript J. Those actuators and sensors located at unconnected points (or located at "interior" points of each substructure) are denoted with a superscript I. To describe interface compatibility and equilibrium conditions in terms of input and output vectors, it is assumed that there are a pair of collocated actuator and sensor at every joint degree of freedom. Also, the output measurements at the joint coordinates are assumed to be accelerations.

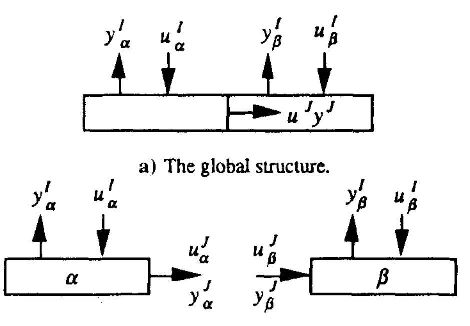
Let the state-space models of α substructure and β substructure be described, respectively, by

$$\begin{aligned} \dot{x}_\alpha &= A_\alpha x_\alpha + [B^I_\alpha \; B^J_\alpha] \begin{Bmatrix} u^I_\alpha \\ u^J_\alpha \end{Bmatrix} \\ \begin{Bmatrix} y^I_\alpha \\ y^J_\alpha \end{Bmatrix} &= \begin{bmatrix} C^I_\alpha \\ C^J_\alpha \end{bmatrix} x_\alpha + \begin{bmatrix} D^{II}_\alpha & D^{IJ}_\alpha \\ D^{JI}_\alpha & D^{JJ}_\alpha \end{bmatrix} \begin{Bmatrix} u^I_\alpha \\ u^J_\alpha \end{Bmatrix} \end{aligned} \tag{1}$$

and

$$\begin{aligned} \dot{x}_\beta &= A_\beta x_\beta + [B^I_\beta \; B^J_\beta] \begin{Bmatrix} u^I_\beta \\ u^J_\beta \end{Bmatrix} \\ \begin{Bmatrix} y^I_\beta \\ y^J_\beta \end{Bmatrix} &= \begin{bmatrix} C^I_\beta \\ C^J_\beta \end{bmatrix} x_\beta + \begin{bmatrix} D^{II}_\beta & D^{IJ}_\beta \\ D^{JI}_\beta & D^{JJ}_\beta \end{bmatrix} \begin{Bmatrix} u^I_\beta \\ u^J_\beta \end{Bmatrix} \end{aligned} \tag{2}$$

These two substructure state-space models can be determined from system identification of substructure experimental data.[8]



a) The global structure.

b) The two substructures.

Fig. 1 Two-substructure structure.

Besides that, they also can be derived from finite element modeling of the two substructures. For instance, let the finite element model of α substructure be described by

$$\begin{aligned} M_\alpha \begin{Bmatrix} \ddot{w}^I_\alpha \\ \ddot{w}^J_\alpha \end{Bmatrix} + Z_\alpha \begin{Bmatrix} \dot{w}^I_\alpha \\ \dot{w}^J_\alpha \end{Bmatrix} + K_\alpha \begin{Bmatrix} w^I_\alpha \\ w^J_\alpha \end{Bmatrix} &= \begin{bmatrix} P^I_\alpha & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} u^I_\alpha \\ u^J_\alpha \end{Bmatrix} \\ \begin{Bmatrix} y^I_\alpha \\ y^J_\alpha \end{Bmatrix} &= \begin{bmatrix} H^I_\alpha & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{w}^I_\alpha \\ \ddot{w}^J_\alpha \end{Bmatrix} \end{aligned} \tag{3}$$

where w^I_α and w^J_α are the interior and the joint displacement coordinates, respectively. M_α, Z_α, and K_α are the substructure mass, damping, and stiffness matrices. P^I_α is the interior actuator distribution matrix. H^I_α is the interior accelerometer distribution matrix. (For more general cases, the interior outputs may also include displacement, velocity, and strain gauge measurements.) The identity matrix I in the input and output matrices indicates that there are collocated actuator and accelerometer at every joint degree of freedom. In first-order state-space form, the system in Eq. (3) can be represented by

$$\begin{aligned} \dot{x}_\alpha &= \begin{bmatrix} 0 & I \\ -M_\alpha^{-1}K_\alpha & -M_\alpha^{-1}Z_\alpha \end{bmatrix} x_\alpha \\ &\quad + \begin{bmatrix} 0 & 0 \\ M_\alpha^{-1}\begin{bmatrix} P^I_\alpha \\ 0 \end{bmatrix} & M_\alpha^{-I}\begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix} \begin{Bmatrix} u^I_\alpha \\ u^J_\alpha \end{Bmatrix} \\ \begin{Bmatrix} y^I_\alpha \\ y^J_\alpha \end{Bmatrix} &= \begin{bmatrix} H^I_\alpha & 0 \\ 0 & I \end{bmatrix} [-M_\alpha^{-1}K_\alpha \; -M_\alpha^{-1}Z_\alpha^{-1}]\, x_\alpha \\ &\quad + \begin{bmatrix} H^I_\alpha & 0 \\ 0 & I \end{bmatrix} M_\alpha^{-1} \begin{bmatrix} P^I_\alpha & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} u^I_\alpha \\ u^J_\alpha \end{Bmatrix} \end{aligned} \tag{4}$$

which has the same form as Eq. (1). The state vector in Eq. (4) is defined by

$$x_\alpha = \begin{Bmatrix} w^I_\alpha \\ w^J_\alpha \\ \dot{w}^I_\alpha \\ \dot{w}^J_\alpha \end{Bmatrix}$$

For convenience of derivation later, we can further rearrange the substructure state vector as

$$x_\alpha = \begin{Bmatrix} w^I_\alpha \\ \dot{w}^I_\alpha \\ w^J_\alpha \\ \dot{w}^J_\alpha \end{Bmatrix} \equiv \begin{Bmatrix} x^I_\alpha \\ x^J_\alpha \end{Bmatrix} \tag{5}$$

and also rearrange the system matrices accordingly.

It should be pointed out here that the inclusion of the analytical models in Eqs. (3) and (4) in this paper is only for derivation purpose. In general, the substructure state vectors, x_α and x_β, can be in any coordinates besides physical coordinates. For instance, if the substructure models are identified from experimental data, then the state vectors are generally not in physical coordinates. Consequently, decomposition of the substructure state vectors into interior and joint portions as in Eq. (5) is not possible. It is for this reason that instead of the state vectors we will use the output vectors y_α and y_β, which measure substructure's motion in physical coordinates, to describe and enforce the compatibility conditions between substructures.

The two state-space models in Eqs. (1) and (2) describe the dynamics of the two substructures when they are completely decoupled. When the two substructures are joined together to form the global structure, their input and output vectors at the interface are no longer independent. The output vectors at the interface are constrained by the compatibility equation

$$y_\alpha^J = y_\beta^J = y^J \tag{6}$$

which says that the physical motion of the two substructures at the interface must be the same. The input vectors at the interface are related by the equilibrium equation

$$u^J = u_\alpha^J = u_\beta^J \tag{7}$$

which says that the sum of the internal forces at the interface must be equal to the external force applied there. Using the compatibility and equilibrium conditions, we can couple the two substructure state-space models. First, rewrite the bottom part of the output equations in Eqs. (1) and (2) as

$$\begin{aligned} u_\alpha^J &= (D_\alpha^{JJ})^{-1}(y_\alpha^J - C_\alpha^J x_\alpha - D_\alpha^{JI} u_\alpha^I) \\ u_\beta^J &= (D_\beta^{JJ})^{-1}(y_\beta^J - C_\beta^J x_\beta - D_\beta^{JI} u_\beta^I) \end{aligned} \tag{8}$$

Adding Eqs. (8) and applying the equilibrium condition in Eq. (7), we obtain

$$u^J = [(D_\alpha^{JJ})^{-1} + (D_\beta^{JJ})^{-1}]\, y^J - (D_\alpha^{JJ})^{-1}(C_\alpha^J x_\alpha + D_\alpha^{JI} u_\alpha^I) - (D_\beta^{JJ})^{-1}(C_\beta^J x_\beta + D_\beta^{JI} u_\beta^I)$$

The preceding equation can be rewritten as

$$y^J = D_\alpha^{JJ} S^{-1} D_\beta^{JJ}[u_\alpha^J + u_\beta^J + (D_\alpha^{JJ})^{-1}(C_\alpha^J x_\alpha + D_\alpha^{JI} u_\alpha^I) + (D_\beta^{JJ})^{-1}(C_\beta^J x_\beta + D_\beta^{JI} u_\beta^I)] \tag{9}$$

where $S = (D_\alpha^{JJ} + D_\beta^{JJ})$ and u^J is replaced by $u_\alpha^J + u_\beta^J$. Finally, by substituting Eq. (9) into Eqs. (8) and then substituting the results into the two substructure state-space models in Eqs. (1) and (2), we obtain the following "coupled" state-space model for the two substructures:

$$\begin{Bmatrix} \dot{x}_\alpha \\ \dot{x}_\beta \end{Bmatrix} = \begin{bmatrix} A_\alpha - B_\alpha^J S^{-1} C_\alpha^J & B_\alpha^J S^{-1} C_\beta^J \\ B_\beta^J S^{-1} C_\alpha^J & A_\beta - B_\beta^J S^{-1} C_\beta^J \end{bmatrix} \begin{Bmatrix} x_\alpha \\ x_\beta \end{Bmatrix} + \left[\begin{array}{c|c|c|c} B_\alpha^I - B_\alpha^J S^{-1} D_\alpha^{JI} & B_\alpha^J S^{-1} D_\beta^{JJ} & B_\alpha^J S^{-1} D_\beta^{JJ} & B_\alpha^J S^{-1} D_\beta^{JI} \\ B_\beta^J S^{-1} D_\alpha^{JI} & B_\beta^J S^{-1} D_\alpha^{JJ} & B_\beta^J S^{-1} D_\alpha^{JJ} & B_\beta^I - B_\beta^J S^{-1} D_\beta^{JI} \end{array}\right] \times \begin{Bmatrix} u_\alpha^I \\ u_\alpha^J \\ u_\beta^J \\ u_\beta^I \end{Bmatrix}$$

$$\begin{Bmatrix} y_\alpha^I \\ y_\alpha^J \\ y_\beta^J \\ y_\beta^I \end{Bmatrix} = \begin{bmatrix} C_\alpha^I - D_\alpha^{IJ} S^{-1} C_\alpha^J & D_\alpha^{IJ} S^{-1} C_\beta^J \\ D_\beta^{JJ} S^{-1} C_\alpha^J & D_\alpha^{JJ} S^{-1} C_\beta^J \\ D_\beta^{JJ} S^{-1} C_\alpha^J & D_\alpha^{JJ} S^{-1} C_\beta^J \\ D_\beta^{IJ} S^{-1} C_\alpha^J & C_\beta^I - D_\beta^{IJ} S^{-1} C_\beta^J \end{bmatrix} \begin{Bmatrix} x_\alpha \\ x_\beta \end{Bmatrix} \tag{10}$$

$$+ \left[\begin{array}{c|c|c|c} D_\alpha^{II} - D_\alpha^{IJ} S^{-1} D_\alpha^{JI} & D_\alpha^{IJ} S^{-1} D_\beta^{JJ} & D_\alpha^{IJ} S^{-1} D_\beta^{JJ} & D_\alpha^{IJ} S^{-1} D_\beta^{JI} \\ D_\beta^{JJ} S^{-1} D_\alpha^{JI} & D_\alpha^{JJ} S^{-1} D_\beta^{JJ} & D_\alpha^{JJ} S^{-1} D_\beta^{JJ} & D_\alpha^{JJ} S^{-1} D_\beta^{JI} \\ D_\beta^{JJ} S^{-1} D_\alpha^{JI} & D_\alpha^{JJ} S^{-1} D_\beta^{JJ} & D_\alpha^{JJ} S^{-1} D_\beta^{JJ} & D_\alpha^{JJ} S^{-1} D_\beta^{JI} \\ D_\beta^{IJ} S^{-1} D_\alpha^{JI} & D_\beta^{IJ} S^{-1} D_\alpha^{JJ} & D_\beta^{IJ} S^{-1} D_\alpha^{JJ} & D_\beta^{II} - D_\beta^{IJ} S^{-1} D_\beta^{JI} \end{array}\right] \times \begin{Bmatrix} u_\alpha^I \\ u_\alpha^J \\ u_\beta^J \\ u_\beta^I \end{Bmatrix}$$

In short notation, Eq. (10) will be represented by

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}\tilde{u} \\ \tilde{y} &= \tilde{C}\tilde{x} + \tilde{D}\tilde{u} \end{aligned} \tag{11}$$

where $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{x}$, $\tilde{u}$, and $\tilde{y}$ are clearly defined.

The state-space model in Eq. (10) will be referred to as the coupled-substructure state-space model. This model describes the dynamics of the two substructures in Fig. 1b with compatibility and equilibrium conditions satisfied at the interface. One can consider the coupled-structure state-space model as a system obtained by connecting the two substructures at the interface by using very short, massless, rigid links. The coupled substructure state-space model has exactly the same input-output transfer function as the global structure in Fig. 1a. Therefore, it can be used as a basis for dynamic simulation and control design for the global structure. In fact, there is a relationship between the coupled-substructure state-space model and the state-space model of the global structure. Let the state-space model of the global structure be described by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{12}$$

with the state, input, and output vectors defined by

$$x = \begin{Bmatrix} x_\alpha^I \\ x^J \\ x_\beta^I \end{Bmatrix}, \quad u = \begin{Bmatrix} u_\alpha^I \\ u^J \\ u_\beta^I \end{Bmatrix}, \quad y = \begin{Bmatrix} y_\alpha^I \\ y^J \\ y_\beta^I \end{Bmatrix} \tag{13}$$

We see that $\tilde{x}$ and x are related as

$$\tilde{x} \equiv \begin{Bmatrix} x_\alpha^I \\ x_\alpha^J \\ x_\beta^I \\ x_\beta^J \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{bmatrix} \begin{Bmatrix} x_\alpha^I \\ x^J \\ x_\beta^I \end{Bmatrix} \quad \text{or} \quad \tilde{x} = Tx \tag{14}$$

where T will be referred to as the state coupling matrix. Also, $\tilde{u}$ and u are related by

$$\begin{Bmatrix} u_\alpha^I \\ u^J \\ u_\beta^I \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{Bmatrix} u_\alpha^I \\ u_\alpha^J \\ u_\beta^J \\ u_\beta^I \end{Bmatrix} \quad \text{or} \quad u = T_1 \tilde{u} \tag{15}$$

and $\tilde{y}$ and y are related by

$$\begin{Bmatrix} y_\alpha^I \\ y_\alpha^J \\ y_\beta^J \\ y_\beta^I \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{Bmatrix} y_\alpha^I \\ y^J \\ y_\beta^I \end{Bmatrix} \quad \text{or} \quad \tilde{y} = T_2 y \tag{16}$$

T_1 and T_2 will be referred to as the input coupling matrix and the output coupling matrix, respectively. It should be noted that in general $T_1 \neq T_2^T$ because the number of interior inputs and the number of interior outputs can be different. By using the state, input, and output coupling matrices, it can be shown that the global structure state-space model and the coupled-substructure state-space model are related by

$$TA = \tilde{A}T, \quad TBT_1 = \tilde{B}, \quad T_2C = \tilde{C}T, \quad T_2DT_1 = \tilde{D} \tag{17}$$

The relations just given will prove to be useful in later derivations. It should be noted that for the relation in Eq. (14) to be realizable, the state vectors must be in physical coordinates. If this is not the case, theoretically there still exists a coupling matrix T that relates the substructure state vectors to the global structure state vector, except that this coupling matrix may not be known.

In the preceding derivation, we have assumed that both D_α^{JJ} and D_β^{JJ} are invertible [Eq. (8)] and so is S [Eq. (10)]. When the interface outputs are accelerations, we get from Eq. (4)

$$D_\alpha^{JJ} = [0\ I]\, M_\alpha^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix}$$

which is positive definite and invertible if M_α is a positive-definite matrix. For most structures, the mass matrix is positive definite and, thus, the assumption is reasonable. Also, $S(= D_\alpha^{JJ} + D_\beta^{JJ})$ is invertible because the sum of two positive-definite matrices is also positive definite.

In the following sections, a method for designing decentralized controllers for the global structure will be derived. It will be shown that the control gains for joint inputs and the observer gains for joint outputs can be chosen such that the coupling between substructures is eliminated.

Design of Decentralized Control Gain Matrix

According to the separation principle, the control gain matrix and the observer gain matrix can be designed separately. In this section, we will develop the procedure for designing decentralized (or localized) state feedback gains for the global structure. The derivation is based on the coupled-substructure state-space model.

The key idea behind the decentralization method presented here is to use the joint actuators to eliminate the interaction between substructures. Let the joint actuator commands be chosen as

$$\begin{aligned} u_\alpha^J &= -(D_\alpha^{JJ})^{-1}(C_\alpha^J x_\alpha + D_\alpha^{JI} u_\alpha^I) \\ u_\beta^J &= -(D_\beta^{JJ})^{-1}(C_\beta^J x_\beta + D_\beta^{JI} u_\beta^I) \end{aligned} \tag{18}$$

Then, after algebraic manipulations, substitution of Eq. (18) into the coupled-substructure state equation in Eq. (10) yields

$$\begin{Bmatrix} \dot{x}_\alpha \\ \dot{x}_\beta \end{Bmatrix} = \begin{bmatrix} A_{\alpha m} & 0 \\ 0 & A_{\beta m} \end{bmatrix} \begin{Bmatrix} x_\alpha \\ x_\beta \end{Bmatrix} + \begin{bmatrix} B_{\alpha m}^I & 0 \\ 0 & B_{\beta m}^I \end{bmatrix} \begin{Bmatrix} u_\alpha^I \\ u_\beta^I \end{Bmatrix} \tag{19}$$

where

$$\begin{aligned} A_{\alpha m} &= A_\alpha - B_\alpha^J (D_\alpha^{JJ})^{-1} C_\alpha^J \\ B_{\alpha m}^I &= B_\alpha^I - B_\alpha^J (D_\alpha^{JJ})^{-1} D_\alpha^{JI} \\ A_{\beta m} &= A_\beta - B_\beta^J (D_\beta^{JJ})^{-1} C_\beta^J \\ B_{\beta m}^I &= B_\beta^I - B_\beta^J (D_\beta^{JJ})^{-1} D_\beta^{JI} \end{aligned} \tag{20}$$

The subscript m is used to denote that the matrices are modified substructure matrices. Note that the system in Eq. (19) is decoupled. This decoupling of the closed-loop state equation suggests a decentralized control design scheme, which can be described as follows. First, restrict the feedback commands for the interior actuators to be localized state feedback as described by

$$u_\alpha^I = G_\alpha^I x_\alpha, \qquad u_\beta^I = G_\beta^I x_\beta \tag{21}$$

where the gain matrices G_α^I and G_β^I are to be designed based on the following modified substructure state equations:

$$\begin{aligned} \dot{x}_\alpha &= A_{\alpha m} x_\alpha + B_{\alpha m}^I u_\alpha^I \\ \dot{x}_\beta &= A_{\beta m} x_\beta + B_{\beta m}^I u_\beta^I \end{aligned} \tag{22}$$

Then, according to Eq. (18), set the feedback commands for joint actuators to be

$$\begin{aligned} u_\alpha^J &= -(D_\alpha^{JJ})^{-1}(C_\alpha^J + D_\alpha^{JI} G_\alpha^I)\, x_\alpha \equiv G_\alpha^J x_\alpha \\ u_\beta^J &= -(D_\beta^{JJ})^{-1}(C_\beta^J + D_\beta^{JI} G_\beta^I)\, x_\beta \equiv G_\beta^J x_\beta \end{aligned} \tag{23}$$

Finally, substitute the actuator commands in Eqs. (21) and (23) into Eq. (15) and use the input coupling matrix to obtain a global feedback gain for control implementation on the global structure.

For the decentralized control design approach just described to be successful, both modified substructure systems ($A_{\alpha m}$, $B_{\alpha m}^I$) and ($A_{\beta m}$, $B_{\beta m}^I$) must be controllable or, at least, all of the unstable poles of $A_{\alpha m}$ and $A_{\beta m}$ need to be controllable. However, this is not the case. Substitution of the joint actuator commands in Eq. (18) into Eq. (8) yields $y_\alpha^J \equiv \ddot{w}_\alpha^J = 0$ and $y_\beta^J \equiv \ddot{w}_\beta^J = 0$, which indicates that all of the joint degrees of freedom are free of forces. Mathematically, the joint actuator commands in Eq. (18) are equal to the internal forces exerting on the joint degrees of freedom but in opposite sign. The joint degrees of freedom force free because all of the forces acting on the joint degrees of freedom, induced by interior inputs or by vibration of substructures, are cancelled out by the joint actuator commands. Consequently, the interaction between the two substructures is eliminated. The only problem is that the joint degrees of freedom, although free of forces, still can have rigid body motion, which is not controllable by interior actuators since all of the control forces coming from interior actuators to the joint degrees of freedom will be cancelled by the joint actuator commands. As a result, the global structure closed-loop system attains a pair of unstable double poles at the origin for every joint degree of freedom. A simple approach to control rigid body motion of the joint degrees of freedom is to introduce an augmented joint actuator command. Details about the design of interior actuator feedback gain matrices, G_α^I and G_β^I, and the design of augmented joint actuator command are described in the following.

In physical coordinates, the modified state equation for α substructure can be partitioned as

$$\begin{Bmatrix} \dot{x}_\alpha^I \\ \dot{x}_\alpha^J \end{Bmatrix} = \begin{bmatrix} A_{\alpha m}^{II} & A_{\alpha m}^{IJ} \\ A_{\alpha m}^{JI} & A_{\alpha m}^{JJ} \end{bmatrix} \begin{Bmatrix} x_\alpha^I \\ x_\alpha^J \end{Bmatrix} + \begin{bmatrix} B_{\alpha m}^{II} \\ B_{\alpha m}^{JI} \end{bmatrix} u_\alpha^I \tag{24}$$

in which the uncontrollable state is x_α^J. ($A_{\alpha m}^{II}$, $B_{\alpha m}^{II}$) is a completely controllable system, presuming that the substructure is controllable by the given set of interior actuators. We can set

$$u_\alpha^I = G_\alpha^I x_\alpha = [G_\alpha^{II}\ 0] \begin{Bmatrix} x_\alpha^I \\ x_\alpha^J \end{Bmatrix} \tag{25}$$

with G_α^{II} to be designed based on the subsystem $\dot{x}_\alpha^I = A_{\alpha m}^{II} x_\alpha^I + B_{\alpha m}^{II} u_\alpha^I$. Any existing state feedback design method, e.g., the LQR control theory or pole placement method, can be used to determine G_α^{II}. Similarly, we can set $G_\beta^I = [G_\beta^{II}\ 0]$ and let G_β^{II} be designed based on $\dot{x}_\beta^I = A_{\beta m}^{II} x_\beta^I + B_{\beta m}^{II} u_\beta^I$. By doing so, the interior degrees of freedom of the structure are controlled and stabilized by u_α^I and u_β^I. If the substructure state-space models are identified from experimental data rather than derived from finite element modeling, then the partition of substructures' modified state equations as in Eq. (24) can not be done because the states are generally not in physical coordinates. In this case, existing system realization algorithms should be used to separate the substructures' modified state equations into controllable subsystems and uncontrollable subsystems. Then, G_α^{II} and G_β^{II} should be designed based on the controllable subsystems.

To control rigid-body motions of the joint degrees of freedom, augmented joint actuator commands are introduced. Let

$$\begin{aligned} u_\alpha^J &= G_\alpha^J x_\alpha + \bar{u}_\alpha^J \\ u_\beta^J &= G_\beta^J x_\beta + \bar{u}_\beta^J \end{aligned} \tag{26}$$

where $\bar{u}_\alpha^J$ and $\bar{u}_\beta^J$ are the augmented joint inputs. Substitution of Eq. (26) into Eq. (8) yields

$$\bar{u}_\alpha^J = (D_\alpha^{JJ})^{-1} y_\alpha^J \equiv (D_\alpha^{JJ})^{-1} \ddot{w}_\alpha^J$$
$$\bar{u}_\beta^J = (D_\beta^{JJ})^{-1} y_\beta^J \equiv (D_\beta^{JJ})^{-1} \ddot{w}_\beta^J \tag{27}$$

We can set the augmented joint inputs to be

$$\bar{u}_\alpha^J = (D_\alpha^{JJ})^{-1} [-[\Omega_i^2] \; -[2\zeta_i\Omega_i]] \begin{Bmatrix} w_\alpha^J \\ \dot{w}_\alpha^J \end{Bmatrix} \equiv \bar{G}_\alpha^{JJ} x_\alpha^J$$
$$\bar{u}_\beta^J = (D_\beta^{JJ})^{-1} [-[\Omega_i^2] \; -[2\zeta_i\Omega_i]] \begin{Bmatrix} w_\beta^J \\ \dot{w}_\beta^J \end{Bmatrix} \equiv \bar{G}_\beta^{JJ} x_\beta^J \tag{28}$$

where $[2\zeta_i\Omega_i]$ and $[\Omega_i^2]$ are diagonal matrices with the design parameters ζ_i and Ω_i, $i = 1, 2, \ldots, n_c$, on the diagonal. Then, substitution of Eq. (28) into Eq. (27) yields the following closed-loop equations for the joint degrees of freedom:

$$\ddot{w}_\alpha^J + [2\zeta_i\Omega_i]\,\dot{w}_\alpha^J + [\Omega_i^2] w_\alpha^J = 0$$
$$\ddot{w}_\beta^J + [2\zeta_i\Omega_i]\,\dot{w}_\beta^J + [\Omega_i^2] w_\beta^J = 0 \tag{29}$$

which show that the closed-loop poles associated with joint degrees of freedom are stable poles located at $-\zeta_i\Omega_i \pm j\Omega_i\sqrt{1-\zeta_i^2}$, $i = 1, 2, \ldots, n_c$. The augmented joint actuator input commands in Eq. (28) can further be expressed as

$$\bar{u}_\alpha^J = [0 \; \bar{G}_\alpha^{JJ}] \begin{Bmatrix} x_\alpha^I \\ x_\alpha^J \end{Bmatrix} \equiv \bar{G}_\alpha^J x_\alpha$$
$$\bar{u}_\beta^J = [0 \; \bar{G}_\beta^{JJ}] \begin{Bmatrix} x_\beta^I \\ x_\beta^J \end{Bmatrix} \equiv \bar{G}_\beta^J x_\beta \tag{30}$$

It is necessary to show that the introduction of augmented joint actuator commands does not affect stability or performance of the closed-loop system. In physical coordinates, the modified α-substructure state equation plus the augmented joint input, i.e., $\dot{x}_\alpha = A_{\alpha m} x_\alpha + B_{\alpha m}^I u_\alpha^I + B_\alpha^J \bar{u}_\alpha^J$ takes the form

$$\begin{Bmatrix} \dot{w}_\alpha^I \\ \ddot{w}_\alpha^I \\ \cdots \\ \dot{w}_\alpha^J \\ \ddot{w}_\alpha^J \end{Bmatrix} = \begin{bmatrix} 0 & I & \vdots & 0 & 0 \\ \times & \times & \vdots & \times & \times \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0 & I \\ 0 & 0 & \vdots & 0 & 0 \end{bmatrix} \begin{Bmatrix} w_\alpha^I \\ \dot{w}_\alpha^I \\ \cdots \\ w_\alpha^J \\ \dot{w}_\alpha^J \end{Bmatrix} + \begin{bmatrix} 0 \\ \times \\ \cdots \\ 0 \\ 0 \end{bmatrix} u_\alpha^I + \begin{bmatrix} 0 \\ \times \\ \cdots \\ 0 \\ \times \end{bmatrix} \bar{u}_\alpha^J \tag{31}$$

in which × denotes nonzero partitions. The form just given is obtained from identifying nonzero terms in A_α, B_α^I, B_α^J, and C_α^J matrices and by using the definitions in Eq. (20). In the notation of Eq. (24), Eq. (31) can be expressed as

$$\begin{Bmatrix} \dot{x}_\alpha^I \\ \dot{x}_\alpha^J \end{Bmatrix} = \begin{bmatrix} A_{\alpha m}^{II} & A_{\alpha m}^{IJ} \\ 0 & A_{\alpha m}^{JJ} \end{bmatrix} \begin{Bmatrix} x_\alpha^I \\ x_\alpha^J \end{Bmatrix} + \begin{bmatrix} B_{\alpha m}^{II} \\ 0 \end{bmatrix} u_\alpha^I + \begin{bmatrix} B_\alpha^{IJ} \\ B_\alpha^{JJ} \end{bmatrix} \bar{u}_\alpha^J \tag{32}$$

The preceding equation clearly shows that the joint degrees of freedom are not controllable by the interior actuators u_α^I because $B_{\alpha m}^{JI} = 0$ and $A_{\alpha m}^{JI} = 0$. Substituting u_α^I of Eq. (25) and $\bar{u}_\alpha^J$ of Eq. (30) into Eq. (32) yields a closed-loop system whose poles are the union of eigenvalues of $(A_{\alpha m}^{II} + B_{\alpha m}^{II} G_\alpha^{II})$, which are closed-loop poles of the interior degrees of freedom, and eigenvalues of $(A_{\alpha m}^{JJ} + B_\alpha^{JJ} \bar{G}_\alpha^{JJ})$, which are equal to the assigned joint degrees-of-freedom closed-loop poles $-\zeta_i\Omega_i \pm j\Omega_i\sqrt{1-\zeta_i^2}$, $i = 1, 2, \ldots, n_c$. Equation (32) also shows that the decoupling between joint degrees of freedom and interior degrees of freedom is only one way. Although the joint degrees of freedom are completely decoupled from the interior degrees of freedom, the interior degrees of freedom are affected by the dynamics of the joint degrees of freedom through the nonzero $A_{\alpha m}^{IJ}$ and B_α^{IJ}. This one way interaction, although it does not affect stability, can degrade the closed-loop system performance. To achieve satisfactory closed-loop performance, the design parameters ζ_i and Ω_i should be chosen such that vibration of the joint degrees of freedom decays much faster than that of the interior degrees of freedom.

In summary, the procedure for designing decentralized control gain matrix consists of three steps. First, the feedback commands for interior actuators are set to be $u_\alpha^I = G_\alpha^I x_\alpha \equiv [G_\alpha^{II} \; 0]\, x_\alpha$ and $u_\beta^I = G_\beta^I x_\beta \equiv [G_\beta^{II} \; 0]\, x_\beta$ with G_α^{II} and G_β^{II} being designed based on $\dot{x}_\alpha^I = A_{\alpha m}^{II} x_\alpha^I + B_{\alpha m}^{II} u_\alpha^I$ and $\dot{x}_\beta^I = A_{\beta m}^{II} x_\beta^I + B_{\beta m}^{II} u_\beta^I$. Second, the feedback commands for the joint actuators are set to be $u_\alpha^J = (G_\alpha^J + \bar{G}_\alpha^J) x_\alpha$ and $u_\beta^J = (G_\beta^J + \bar{G}_\beta^J) x_\beta$, where G_α^J and G_β^J are defined in Eq. (23) and $\bar{G}_\alpha^J$ and $\bar{G}_\beta^J$ are defined in Eqs. (28) and (30). This set of feedback commands can be summarized by one equation

$$\begin{Bmatrix} u_\alpha^I \\ u_\alpha^J \\ u_\beta^J \\ u_\beta^I \end{Bmatrix} = \begin{bmatrix} G_\alpha^I & 0 \\ G_\alpha^J + \bar{G}_\alpha^J & 0 \\ 0 & G_\beta^J + \bar{G}_\beta^J \\ 0 & G_\beta^I \end{bmatrix} \begin{Bmatrix} x_\alpha \\ x_\beta \end{Bmatrix} \quad \text{or} \quad \tilde{u} = \tilde{G}\tilde{x} \tag{33}$$

Finally, after the two substructures are connected together, a global feedback gain matrix G for the global structure can be obtained by substituting Eq. (33) into Eq. (15). The result is

$$u = T_1\tilde{u} = T_1\tilde{G}\tilde{x} \equiv G\tilde{x} \tag{34}$$

The closed-loop poles of the joint degrees of freedom are not changed by the interconnection of substructures as long as the same values of Ω_i and ζ_i are assigned to each pair of connecting joint degrees of freedom. The set of feedback commands derived in this section will be called the joint decoupling actuator commands.

Design of Decentralized Observers

In most practical situations, the number of sensors is limited and thus full measurement of the system's state is not available. To implement state feedback control, an observer is required to reconstruct the entire system state. In this section, the procedure for designing decentralized observers is developed.

The derivation is again based on the coupled-substructure state-space model in Eq. (10), whose short notation expression is given by Eq. (11). Let the observer be described by

$$\dot{\tilde{q}} = \tilde{A}\tilde{q} + \tilde{B}\tilde{u} - \tilde{F}(\tilde{y} - \tilde{C}\tilde{q} - \tilde{D}\tilde{u}) \tag{35}$$

where

$$\tilde{q} \equiv \begin{Bmatrix} q_\alpha \\ q_\beta \end{Bmatrix}$$

is the observer state vector with q_α being the estimate of x_α and q_β being the estimate of x_β, respectively. $\tilde{F}$ is the observer gain matrix to be designed. According to Eq. (33), state feedback control using estimated state is done by setting

$$\tilde{u} = \tilde{G}\tilde{q} = \begin{bmatrix} G_\alpha^I & 0 \\ G_\alpha^J + \bar{G}_\alpha^J & 0 \\ 0 & G_\beta^J + \bar{G}_\beta^J \\ 0 & G_\beta^I \end{bmatrix} \begin{Bmatrix} q_\alpha \\ q_\beta \end{Bmatrix} \tag{36}$$

Substituting Eq. (36) into the observer equation (35) and performing algebraic manipulation, we get

$$\begin{Bmatrix} \dot{q}_\alpha \\ \dot{q}_\beta \end{Bmatrix} = \begin{bmatrix} A_{\alpha m} + B^I_{\alpha m} G^I_\alpha & 0 \\ 0 & A_{\beta m} + B^I_{\beta m} G^I_\beta \end{bmatrix} \begin{Bmatrix} q_\alpha \\ q_\beta \end{Bmatrix} + \begin{bmatrix} B^J_\alpha S^{-1} D^{JJ}_\beta \\ B^J_\beta S^{-1} D^{JJ}_\alpha \end{bmatrix} \bar{u}^J + \tilde{F} \left(\begin{bmatrix} C^I_{\alpha m} + D^{II}_{\alpha m} G^I_\alpha & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & C^I_{\beta m} + D^{II}_{\beta m} G^I_\beta \end{bmatrix} \begin{Bmatrix} q_\alpha \\ q_\beta \end{Bmatrix} + \begin{bmatrix} D^{IJ}_\alpha S^{-1} D^{JJ}_\beta \\ D^{JJ}_\alpha S^{-1} D^{JJ}_\beta \\ D^{JJ}_\alpha S^{-1} D^{JJ}_\beta \\ D^{IJ}_\beta S^{-1} D^{JJ}_\alpha \end{bmatrix} \bar{u}^J - \begin{Bmatrix} y^I_\alpha \\ y^J_\alpha \\ y^J_\beta \\ y^I_\beta \end{Bmatrix} \right) \tag{37}$$

where $\bar{u}^J = (\bar{G}^J_\alpha q_\alpha + \bar{G}^J_\beta q_\beta)$ and

$$\begin{aligned} C^I_{\alpha m} &= C^I_\alpha - D^{IJ}_\alpha (D^{JJ}_\alpha)^{-1} C^J_\alpha \\ D^{II}_{\alpha m} &= D^{II}_\alpha - D^{IJ}_\alpha (D^{JJ}_\alpha)^{-1} D^{JI}_\alpha \\ C^I_{\beta m} &= C^I_\beta - D^{IJ}_\beta (D^{JJ}_\beta)^{-1} C^J_\beta \\ D^{II}_{\beta m} &= D^{II}_\beta - D^{IJ}_\beta (D^{JJ}_\beta)^{-1} D^{JI}_\beta \end{aligned} \tag{38}$$

The matrices just given in addition to the $A_{\alpha m}$, $A_{\beta m}$, $B^I_{\alpha m}$, and $B^I_{\beta m}$ matrices given in Eq. (20) completely define the modified substructure state-space models.

Equation (37) indicates that a decentralized observer design can be achieved by appropriate choice of observer gain matrix. First, restrict the observer gain matrix $\tilde{F}$ to take the decentralized form

$$\tilde{F} = \begin{bmatrix} F^I_\alpha \, F^J_\alpha & 0 & 0 \\ 0 & 0 & F^J_\beta \, F^I_\beta \end{bmatrix} \tag{39}$$

which is similar to the form of the feedback gain matrix $\tilde{G}$ in Eq. (33). Then, set the joint output observer gain matrices to be

$$\begin{aligned} F^J_\alpha &= -(B^J_\alpha + F^I_\alpha D^{IJ}_\alpha)(D^{JJ}_\alpha)^{-1} \\ F^J_\beta &= -(B^J_\beta + F^I_\beta D^{IJ}_\beta)(D^{JJ}_\beta)^{-1} \end{aligned} \tag{40}$$

Substitution of Eqs. (39) and (40) into the observer equation (37) gives

$$\begin{aligned} \dot{q} &= (A_{\alpha m} + B^I_{\alpha m} G^I_\alpha + F^I_\alpha C^I_{\alpha m} + F^I_\alpha D^{II}_{\alpha m} G^I_\alpha)\, q_\alpha - F^I_\alpha y^I_\alpha - F^J_\alpha y^J_\alpha \\ \dot{q} &= (A_{\beta m} + B^I_{\beta m} G^I_\beta + F^I_\beta C^I_{\beta m} + F^I_\beta D^{II}_{\beta m} G^I_\beta)\, q_\beta - F^I_\beta y^I_\beta - F^J_\beta y^J_\beta e \end{aligned} \tag{41}$$

Or, in short notation,

$$\begin{aligned} \dot{q}_\alpha &= E_\alpha q_\alpha - [F^I_\alpha \, F^J_\alpha]\, y_\alpha \\ \dot{q}_\beta &= E_\beta q_\beta - [F^I_\beta \, F^J_\beta]\, y_\beta \end{aligned} \tag{42}$$

where the definitions of E_α and E_β are obvious. The given observer equations are decoupled. The next step is to design the interior output observer gain matrices F^I_α and F^I_β.

Define a state estimation error vector as

$$\tilde{e} \equiv \begin{Bmatrix} e_\alpha \\ e_\beta \end{Bmatrix} \equiv \begin{Bmatrix} x_\alpha - q_\alpha \\ x_\beta - q_\beta \end{Bmatrix} \tag{43}$$

Then, after algebraic manipulations, subtraction of the observer equation (35) from the coupled-substructure state equation in Eq. (11) yields

$$\begin{Bmatrix} \dot{e}_\alpha \\ \dot{e}_\beta \end{Bmatrix} = \begin{bmatrix} A_{\alpha m} + F^I_\alpha C^I_{\alpha m} & 0 \\ 0 & A_{\beta m} + F^I_\beta C \end{bmatrix} \begin{Bmatrix} e_\alpha \\ e_\beta \end{Bmatrix} \tag{44}$$

which is decoupled. Any existing pole placement algorithm can be used to determine F^I_α and F^I_β. A rule of thumb is to choose F^I_α and F^I_β such that eigenvalues of $(A_{\alpha m} + F^I_\alpha C_{\alpha m})$ and $(A_{\beta m} + F^I_\beta C^I_{\beta m})$ are further to the left in the complex plane than the regulator poles of the closed-loop system. Note that we have made an assumption that both $(A_{\alpha m}, C^I_{\alpha m})$ and $(A_{\beta m}, C^I_{\beta m})$ are completely observable. This assumption is by no means a restriction because in practice it is usually possible to arrange locations of interior sensors to meet this requirement.

Although the derivation so far has been based on a two-substructure structure, extension of the method to multisubstructure structure is straightforward. The controller formed by the joint decoupling actuator commands and the substructure observers will be called the joint decoupling controller, whose design procedure is summarized by the following steps.

1) Determine substructure state-space models

$$\dot{x}_s = A_s x_s + [B^I_s \, B^J_s] \begin{Bmatrix} u^I_s \\ u^J_s \end{Bmatrix}, \qquad \begin{Bmatrix} y^I_s \\ y^J_s \end{Bmatrix} = \begin{bmatrix} C^I_s \\ C^J_s \end{bmatrix} x_s + \begin{bmatrix} D^{II}_s \, D^{IJ}_s \\ D^{JI}_s \, D^{JJ}_s \end{bmatrix} \begin{Bmatrix} u^I_s \\ u^J_s \end{Bmatrix} \qquad s = \alpha, \beta, \gamma, \ldots$$

from finite element modeling or from system identification of the substructure experimental data.

2) Derive the modified substructure state-space models by using Eqs. (20) and (38)

$$\dot{x}_s = A_{sm} x_s + B^I_{sm} u^I_s, \qquad y^I_s = C^I_{sm} x_s + D^{II}_{sm} u^I_s \qquad s = \alpha, \beta, \gamma, \ldots$$

3) Design feedback gains for each substructure:
   a) Design G^{II}_s based on $(A^{II}_{sm}, B^{II}_{sm})$ and form $G^I_s = [G^{II}_s \; 0]$.
   b) Set $G^J_s = -(D^{JJ}_s)^{-1}(C^J_s + D^{JI}_s G^I_s)$. Choose appropriate values of ζ_i and Ω_i for joint degrees of freedom and form $\bar{G}^J_s = [0 \; \bar{G}^{JJ}]$, where

$$\bar{G}^{JJ}_s = (D^{JJ}_s)^{-1}[-[\Omega^2_i] - [2\zeta_i \Omega_i]].$$

   c) Set $u^I_s = G^I_s q_s$, $u^J_s = (G^J_s + \bar{G}^J_s) q_s$.

4) Design observer for each substructure:
   d) Design F^I_s based on (A_{sm}, C^I_{sm}).
   e) Set $F^J_s = -(B^J_s + F^I_s D^{IJ}_s)(D^{JJ}_s)^{-1}$.
   f) Form substructure observer equation:

$$\dot{q}_s = (A_{sm} + B^I_{sm} G^I_s + F^I_s C^I_{sm} + F^I_s D^{II}_{sm} G^I_s)\, q_s - F^I_s y^I_s - F^J_s y^J_s$$

5) Form global feedback gain matrix for the global structure by using the input coupling matrix T_1, i.e., by using the relation $u = T_1 \tilde{u}$ [see Eqs. (33) and (34)].

It should be emphasized that although substructure feedback gain matrices need to be assembled at step 5 for implementation on the global structure, there is no need to assemble substructure observers. In fact, substructure observers remain independent of each other at the implementation stage so that the control scheme is truly decentralized. The two substructure observers do share common outputs from the joint sensors, because in actual implementation y^J_α and y^J_β in Eqs. (41) should be replaced by y^J as required by the compatibility condition in Eq. (6). The closed-loop system equation, obtained by combining the global structure state-space model in Eq. (12), the feedback law in Eq. (36), and the substructure observers in Eq. (42), is given by

$$\begin{Bmatrix} \dot{x} \\ \dot{\tilde{q}} \end{Bmatrix} = \begin{bmatrix} A & BT_1\tilde{G} \\ -\tilde{F}T_2C & E - \tilde{F}T_2DT_1\tilde{G} \end{bmatrix} \begin{Bmatrix} x \\ \tilde{q} \end{Bmatrix} \tag{45}$$

where $\tilde{E}$ is a block diagonal matrix formed by E_α and E_β. In terms of the error vector $\tilde{e}$ and by using the relations in Eq. (17), the closed-loop equation (45) can further be converted to

$$\begin{Bmatrix} \dot{x} \\ \dot{\tilde{e}} \end{Bmatrix} = \begin{bmatrix} A + BGT & -BG \\ 0 & \tilde{A} + \tilde{F}\tilde{C} \end{bmatrix} \begin{Bmatrix} x \\ \tilde{e} \end{Bmatrix} \tag{46}$$

The derivation shows that the collection of substructure observers together with the global feedback gain matrix G form a stabilizing controller for the global structure. Another interesting property of the proposed controller is that each individual substructure observer together with substructure feedback gain matrix also constitute a stabilizing controller for the corresponding individual substructure. To be more precise, define α-substructure feedback gain matrix and observer gain matrix to be

$$G_\alpha \equiv \begin{Bmatrix} G_\alpha^I \\ G_\alpha^J + \bar{G}_\alpha^J \end{Bmatrix}, \qquad F_\alpha \equiv [F_\alpha^I \quad F_\alpha^J] \tag{47}$$

Then, by combining the "unconnected" α-substructure model in Eq. (1), the α observer in Eq. (42a) and the feedback laws $u_\alpha^I = G_\alpha^I q_\alpha$ and $u_\alpha^J = (G_\alpha^J + \bar{G}_\alpha^J) q_\alpha$, we obtain the following closed-loop equation:

$$\begin{Bmatrix} \dot{x}_\alpha \\ \dot{q}_\alpha \end{Bmatrix} = \begin{bmatrix} A_\alpha & B_\alpha G_\alpha \\ -F_\alpha C_\alpha & E_\alpha - F_\alpha D_\alpha G_\alpha \end{bmatrix} \begin{Bmatrix} x_\alpha \\ q_\alpha \end{Bmatrix} \tag{48}$$

which can further be converted to

$$\begin{Bmatrix} \dot{x}_\alpha \\ \dot{e}_\alpha \end{Bmatrix} = \begin{bmatrix} A_{\alpha m} + B_{\alpha m}^I G_{\alpha m}^I + B_\alpha^J \bar{G}_\alpha^J & -B_\alpha G_\alpha \\ 0 & A_{\alpha m} + F_\alpha^I C_{\alpha m}^I \end{bmatrix} \begin{Bmatrix} x_\alpha \\ e_\alpha \end{Bmatrix} \tag{49}$$

The closed-loop poles are the same as those of the α substructure when it is connected to β. Therefore, the whole system's closed-loop poles are the same before and after the connection. Besides that, if exact states are available for feedback (which implies the case of full state measurement), then any pair of connecting joint degrees of freedom will have the same response before and after connection, providing that their initial conditions are the same. Consequently, each substructure's closed-loop response is the same whether the substructures are connected or not. When feedback is based on estimated state and when the initial conditions are different, the connecting joint degrees of freedom may not have the same response. Nevertheless, as long as the design criterion sets fast decaying dynamics for joint degrees of freedom, the interconnection will affect the interior degrees of freedom only to a slight extent. This feature makes the joint decoupling controller very attractive for active control of space structures that are required to be connected and disconnected on a routine basis. No controller redesign or controller shut-off-and-turn-on is necessary before, during, and after the entire connecting process.

In practice, the dimensions of substructures may be still too large for control design purposes, and a model reduction step is inevitable. Although model reduction introduces the so-called "spillover" problem, the closed-loop system of the global structure is stable as long as all of the substructure closed-loop systems are stable.

Mass-Spring-Damper Example

As an example, let us consider the mass-spring-damper system shown in Fig. 2. The two substructures α and β are to be connected by using the rigid links on mass 4 and mass 5. Displacement coordinates of the global structure and the two substructures are designated by w_i, $w_{\alpha i}$, and $w_{\beta i}$, respectively. The outputs of the α substructure are $y_\alpha^I = w_{\alpha 1}$ and $y_\alpha^J = \ddot{w}_{\alpha 4}$. The outputs of the β substructure are $y_\beta^J = \ddot{w}_{\beta 3}$ and $y_\beta^I = w_{\beta 1}$. All of the inputs are forces.

The method in this paper is used to design state feedback gain matrices for the two substructures independently. First, the local feedback gain matrices G_α^{II} and G_β^{II} for the interior actuators are determined by solving the following optimization problems:

$$\text{Minimize } J_s = \int_0^\infty (x_s^I)^T Q_s^{II} x_s^I + \rho (u_s^I)^T u_s^I \, dt$$

$$\text{subject to } \dot{x}_s^I = A_{sm}^{II} x_s^I + B_{sm}^{II} u_s^I$$

for $s = \alpha$, β, in which the regulation cost weighting matrix was chosen to be

$$Q_s^{II} = \begin{bmatrix} K_s^{II} & 0 \\ 0 & M_s^{II} \end{bmatrix}$$

such that the first term in the integral represents the sum of strain energy and kinetic energy associated with the interior degrees of freedom. The control cost weight parameter ρ was set to be 0.1. For the augmented joint actuator commands the design parameters were chosen to be $\Omega^2 = 25$ and $2\zeta\Omega = 8$ such that the closed-loop poles of the joint degree of freedom are $-4 \pm j3$. For each substructure, an observer is also designed by using the proposed decentralized observer design procedure. The interior output observer gain matrices F_α^I and F_β^I are determined by using a pole placement method with the criterion that the observer poles, or the eigenvalues of $A_{\alpha m} + F_\alpha^I C_{\alpha m}^I$ and $A_{\beta m} + F_\beta^I C_{\beta m}^I$, are located 12 units to the left of regulator poles in the complex plane. The joint output observer gain matrices F_α^J and F_β^J are determined by Eq. (40).

The design results are summarized in Tables 1 and 2 and Figs. 3–6. Table 1 lists the feedback gain matrices and observer

Table 1 State feedback gain matrices obtained from decentralized control design

Substructure α: $G_\alpha = \begin{bmatrix} G_\alpha^I \\ \underline{G}_\alpha^J + \bar{G}_\alpha^J \end{bmatrix}$, $\quad F_\alpha = \begin{bmatrix} F_\alpha^I \; F_\alpha^J \end{bmatrix}$

$G_\alpha^I = [6.6628 \; -9.2741 \; 2.7544 \; 0.1350 \; -2.9039 \; -2.1168 \; 0 \; 0]$

$\underline{G}_\alpha^J = [0 \; 0 \; -3 \; 0 \; 0 \; 0 \; 3 \; 0]$

$\bar{G}_\alpha^J = [0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 5 \; 1.6]$

$F_\alpha^I = -[0.0001 \; 0.0200 \; 0.2841 \; 1.4080 \; 0.0056 \; 0.3446 \; 1.9427 \; 2.7322]^T (10^6)$

$F_\alpha^J = [0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; -1]^T$

Substructure β: $G_\beta = \begin{bmatrix} G_\beta^I \\ \underline{G}_\beta^J + \bar{G}_\beta^J \end{bmatrix}$, $\quad F_\beta = \begin{bmatrix} F_\beta^I \; F_\beta^J \end{bmatrix}$

$G_\beta^I = [3.9610 \; -7.7589 \; -1.5642 \; -2.2097 \; 0 \; 0]$

$\underline{G}_\beta^J = [0 \; -6 \; 0 \; -0.05 \; 6 \; 0.05]$

$\bar{G}_\beta^J = [5 \; 1.6 \; 0 \; 0 \; 0 \; 0]$

$F_\beta^I = -[1.3976 \; 0.5154 \; 0.0091 \; 3.8874 \; 5.2230 \; 0.3449]^T (10^4)$

$F_\beta^J = [0 \; 0 \; 0 \; -1 \; 0 \; 0]^T$

Global structure: $G = \begin{bmatrix} G_\alpha^I & 0 \\ \underline{G}_\alpha^J + \bar{G}_\alpha^J & \underline{G}_\beta^J + \bar{G}_\beta^J \\ 0 & G_\beta^I \end{bmatrix}$

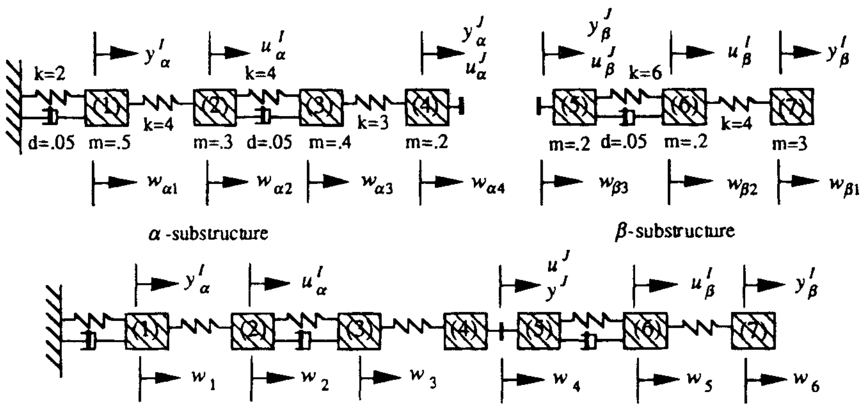


Fig. 2 Mass-spring-damper example.

Table 2 Poles of closed-loop systems

Regular poles	Observer poles
Substructure α	
$-3.0178 \pm j5.6888$	$-15.0178 \pm j5.6888$
$-0.3371 \pm j3.8531$	$-12.3371 \pm j3.8531$
$-1.6808 \pm j2.0713$	$-13.6808 \pm j2.0713$
$-4.0000 \pm j3.0000$	$-16.0000 \pm j3.0000$
Substructure β	
$-4.4318 \pm j7.1943$	$-16.4318 \pm j7.1943$
$-1.2173 \pm j2.7689$	$-13.2173 \pm j2.7689$
$-4.0000 \pm j3.0000$	$-16.0000 \pm j3.0000$
Global structure	
$-3.0178 \pm j5.6888$	$-15.0178 \pm j5.6888$
$-0.3371 \pm j3.8531$	$-12.3371 \pm j3.8531$
$-1.6808 \pm j2.0713$	$-13.6808 \pm j2.0713$
$-4.4318 \pm j7.1943$	$-16.4318 \pm j7.1943$
$-1.2173 \pm j2.7689$	$-13.2173 \pm j2.7689$
$-4.0000 \pm j3.0000$	$-16.0000 \pm j3.0000$
	$-16.0000 \pm j3.0000$

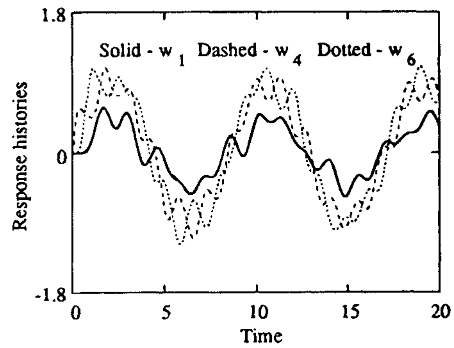


Fig. 3 Open-loop response histories of the global structure.

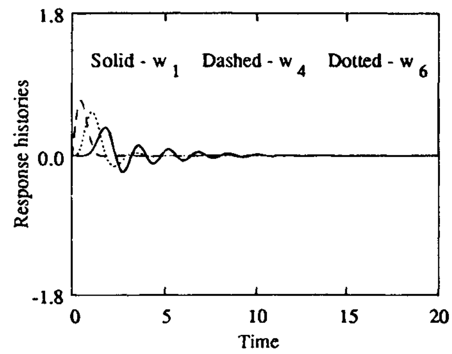


Fig. 4 Closed-loop response histories of the global structure.

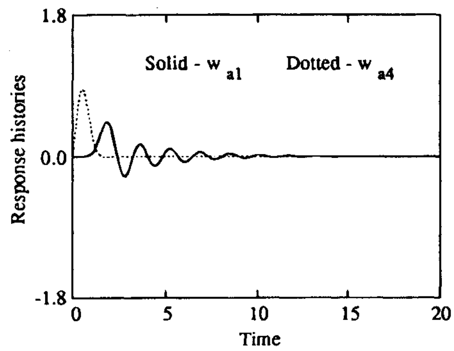


Fig. 5 Closed-loop response histories of the α substructure.

gain matrices of each substructure. The formula of the global structure feedback gain matrix is also given, which is simply an assemblage of substructure feedback gain matrices. Table 2 lists the poles of the closed-loop systems. Each substructure has a pair of regular poles $-4 \pm j3$, which are the closed-loop poles of the joint degree of freedom. The global structure regulator poles are the union of substructure regulator poles, except that one pair of $-4 \pm j3$ are missing because the substructure joints have been connected. Each substructure's observer poles are equal to its regulator poles minus 12 as the design specification required. The global structure observer poles are the union of substructure observer poles, since the global structure observer is nothing but the collection of the two substructure observers.

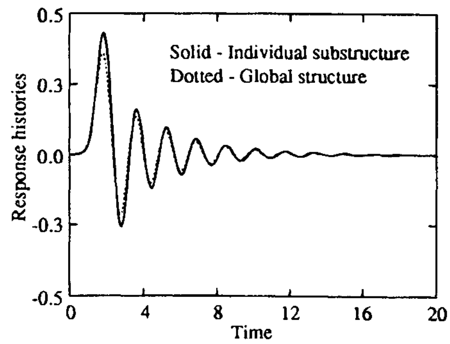


Fig. 6 Comparison of closed-loop response histories of DOF 1.

For response simulation study, let us assume the global structure is impacted by a unit impulse force on the joint degree of freedom at time zero. This impact force gives the joint degree of freedom an initial velocity of magnitude 2.5. Thus, the initial state of the structure is given by $x_0 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 2.5, 0, 0]$. The initial states of the substructure observers, however, were set to be zero because in practice the system's initial state is usually unknown. Based on these initial conditions, the global structure's open-loop and closed-loop displacement response histories at w_1, w_4, and w_6 degrees of freedom are calculated and plotted in Fig. 3 and Fig. 4, respectively. Evidently, the proposed joint decoupling controller suppressed the vibration of the global structure successfully. At the substructure level the controller also works very well, as can be seen from Fig. 5, which shows the closed-loop response histories of α substructure by itself, i.e., when it is not connected with the β substructure. Also, the comparison of closed-loop response of the global structure and closed-loop response of the "disconnected" α substructure in Fig. 6 demonstrates what we mentioned previously, that the interconnection of substructures affects the system's response only slightly. In fact, the two curves in Fig. 6 would match exactly if the substructure observers' initial states were set to be the same as the substructures' initial conditions instead of zero. The slight difference in Fig. 6 is a result of estimated state feedback.

Concluding Remarks

A substructure-based control design procedure using the idea of joint decoupling has been developed for decentralized control of large complex flexible structures. The method assumes there is a pair of collocated actuator and sensor at every joint degree of freedom. Conditions of compatibility and equilibrium at the interface between substructures are used to derive a coupled-substructure state-space model for the structure, which leads to modified substructure state-space models to be used as the decentralized design basis. The design concept is summarized as follows. The control commands for interior actuators of each substructure are chosen to be localized state feedback. Then, the joint actuator commands are set to cancel out all of the forces acting on the joint degrees of freedom, so that the interactions between substructures are eliminated. A similar idea is used in designing decentralized observers. The advantages of the proposed method are 1) the design process is completely decentralized, 2) the controllers can function at both individual substructure level and global structure level, and 3) a large-scale control design problem is divided into several small-scale subproblems. A six-degree-of-freedom mass-spring-damper system is used to demonstrate the proposed method.

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